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CALCULATION OF PRESSURE IN THE LINEAR PROBLEM OF VIBRATOR IN A SUPERSONIC BOUNDARY LAYER

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The supersonic flow over a body consisting of a triangular oscillating plate — the vibrator — mounted between two flat plates is investigated. The body is assumed to be thermally insulated, and the vibrator dimensions and the oscillation frequencies to be such that the flow can be defined by equations of a boundary layer with self-induced pressure [1 — 5]. The oscillation amplitude is assumed small so that these equations can be linearized. The Fourier transform of the longitudinal coordinate is used for solution derivation. The inverse Fourier transform is obtained by numerical methods. It is shown that the perturbations of flow parameters induced by the vibrator are damped upstream and downstream in accordance with an exponential law.

1. Statement of the problem. We consider the flow over a thermally insulated body consisting of a flat plate at rest changing into an oscillating triangular part —the vibrator — and ending with an immovable flat plate (Fig. 1). The length of the front part is L^* and that of the rear part $O(L^*)$ (the asterisk denotes dimensional quantities). The vibrator dimensions are assumed small and will be defined below. The unperturbed oncoming supersonic stream at Mach number M_∞ exceeding unity by a finite quantity flows over the stationary parts of the plate at velocity U_∞. Gas parameters of the unperturbed steady stream and at the wall are denoted by superscripts ∞ and w, respectively. We use the Cartesian system of coordinates x, y with origin at the point of junction of the forward immobile part with the oscillating part. The following notation is used: t^* for time, v_x^* and v_y^* for the velocity vector components, ρ^* for density, p^* for pressure, T^* for temperature, and $\varkappa$ for the specific heat ratio. For simplicity we assume the dependence of the first viscosity coefficient on temperature to be linear, i.e. $\lambda_1^*/\lambda_{1\infty}^* = CT'$, where $T' = T^* / T_\infty^*$, and the Prandtl number to be unity. As the inverse of the Reynolds number we use the small parameter $\varepsilon = \mathrm{Re}_1^{-1/8}$ $(\mathrm{Re}_1 = \rho_\infty^* U_\infty^* L^* / \lambda_{1\infty}^*)$.

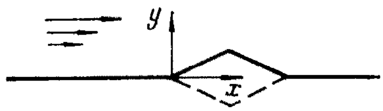

Fig. 1

Let us select the longitudinal dimension of the oscillating part $O(\varepsilon^3)$, the oscillation amplitude $U(\varepsilon^5)$ and frequency $O(\varepsilon^{-2})$. For defining the motion it is convenient to separate three distinct regions [1, 2] : the upper region of the supersonic inviscid flow $(y_1 = O(\varepsilon^3))$, the intermediate region of the conventional boundary layer, $(y_2 = O(\varepsilon^4))$, and the lower region of the boundary layer with self-induced pressure ($y_3 = O(\varepsilon^5)$). The main difficulties of such scheme are related to the construction of solution for the inner region, a solution that makes it possible to obtain the flow parameters in the intermediate and outer regions in explicit form [1 — 5]. Below, we deal only with the inner region, where we introduce the following dimensionless dependent and independent variables [4, 5]:

$$\begin{aligned}
t^* &= L^* U_\infty^{*-1} \varepsilon^2 C^{1/4} \lambda^{-3/2} (M_\infty{}^2 - 1)^{-1/4} T_w' t \\
x^* &= L^* \varepsilon^3 C^{3/8} \lambda^{-5/4} (M_\infty{}^2 - 1)^{-3/8} T_w'^{3/2} x \\
y^* &= L^* \varepsilon^5 C^{5/8} \lambda^{-3/4} (M_\infty{}^2 - 1)^{-1/8} T_w'^{3/2} y \\
v_x^* &= U_\infty{}^* \varepsilon C^{1/8} \lambda^{1/4} (M_\infty{}^2 - 1)^{-1/8} T_w'^{1/2} u \\
v_y^* &= U_\infty{}^* \varepsilon^3 C^{3/8} \lambda^{3/4} (M_\infty{}^2 - 1)^{1/8} T_w'^{1/2} v \\
p^* &= p_\infty{}^* + \rho_\infty{}^* U_\infty^{*2} \varepsilon^2 C^{1/4} \lambda^{1/2} (M_\infty{}^2 - 1)^{-1/4} p \\
\rho^* &= \rho_\infty{}^* T_w' \rho
\end{aligned} \qquad (1.1)$$

The constant $\lambda = 0.3321$ in formulas (1. 1) is defined by the equality $L^* \mathrm{Re}_1{}^{-1/2} \partial (v_x^*/U_\infty) / \partial y^* = \lambda C^{-1/2} T_w'$ in conformity with the Blasious solution for the unperturbed boundary layer. Substituting the expressions (1. 1) into the system of Navier — Stokes equations, retaining principal terms containing ε, and stipulating the fulfilment of conditions of merging with the conventional boundary layer, as $x \to -\infty$ and $y \to \infty$, for the unsteady boundary layer with self-induced pressure [4, 5] we obtain

$$\begin{aligned}
&\rho = 1, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial p}{\partial y} = 0 \\
&\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \\
&x \to -\infty, \quad u \to y, \quad p \to 0 \\
&y \to \infty, \quad u \to y - \int_{-\infty}^{x} p\, dx
\end{aligned} \qquad (1.2)$$

We specify the adhesion conditions at the wall as

$$u = u_w, \quad v = v_w \qquad (1.3)$$

and the oscillating part of the wall by the equation

$$y_w = \sigma f_1(x) \cos \omega t, \quad \sigma \ll 1 \qquad (1.4)$$

where ω is the dimensionless frequency and function $f_1(x)$ (Fig. 1) defines the triangular form with parameters a and b

$$f_1(x) = \begin{cases} 0, & x \leqslant 0 \\ 2x, & 0 \leqslant x \leqslant b \\ 2b(a-x)/(a-b), & b \leqslant x \leqslant a \\ 0, & x \geqslant a \end{cases} \qquad (1.5)$$

The smallness of parameter σ enables us to linearize the problem by expanding the unknown functions in series in powers of σ

$$u = y + \sigma u_1 + \ldots, \quad v = \sigma v_1 + \ldots, \quad p = \sigma p_1 + \ldots$$

whose substitution into (1.2) yields

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \quad \frac{\partial p_1}{\partial y} = 0 \tag{1.6}$$
$$\frac{\partial u_1}{\partial t} + y\frac{\partial u_1}{\partial x} + v_1 = -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2}$$
$$x \to -\infty, \; u_1 \to 0, \; p_1 \to 0$$
$$y \to \infty, \quad u_1 \to -\int_{-\infty}^{x} p_1 \, dx$$

Such linearization is valid everywhere where $u_1 = O(1)$, except in region $y \to 0$, i.e. at the wall where the adhesion conditions (1.3) hold. To obtain conditions for functions u_1 and v_1 we introduce a supplementary subregion with the characteristic variable $y_l = y / \sigma$ in which we seek a solution of the form

$$u = \sigma u_l + \ldots, \quad v = \sigma v_l + \ldots, \quad p = \sigma p_l + \ldots \tag{1.7}$$

Conditions (1.3) at the wall for the newly introduced functions are now of the form

$$u_l(t, x, f_1 \cos \omega t) = 0, \; v_l(t, x, f_1 \cos \omega t) = -\omega f_1 \sin \omega t \tag{1.8}$$

For $y_l \to \infty$ the valid conditions of merging with functions u_1 and v_1 at $y \to 0$ are

$$\sigma u_l(t, x, y_l) \to y + \sigma u_1(t, x, 0) + \ldots \tag{1.9}$$
$$v_l(t, x, y_l) \to v_1(t, x, 0) + \ldots$$

Substituting functions (1.7) into the system of Eqs. (1.2) we obtain

$$\partial v_l / \partial y_l = 0, \quad \partial^2 u_l / \partial y^2{}_l = 0$$

The solution of this very simple system which satisfies conditions (1.8) is of the form

$$u_l = y_l F(t, x) - F(t, x) f_1(x) \cos \omega t$$
$$v_l = -\omega f_1(x) \sin \omega t$$

where $F(t, x)$ is an arbitrary function. From (1.9) we find that $F(t, x) \equiv 1$ and, also, obtain the conditions for functions u_1 and v_1 at $y = 0$

$$u_1(t, x, 0) = -f_1(x) \cos \omega t \tag{1.10}$$
$$v_1(t, x, 0) = -\omega f_1(x) \sin \omega t$$

Problem (1.6), (1.10) was studied in [6], where the unknown functions u_1, v_1, and p_1 were expanded in Fourier integral in variables t and x, but the sought functions were not calculated. The aim of this investigation is to carry out these calculations and, also, to analyze the asymptotic properties of solutions of problem (1.6), (1.10).

2. Calculation of pressure. As in [6], we shall study pressure using equations of the form

$$p_1 = \frac{1}{\pi}\cos\omega t \int_{-\infty}^{\infty} \Phi_1\, d\omega_2 - \frac{1}{\pi}\sin\omega t \int_{-\infty}^{\infty} \Phi_2\, d\omega_2 \tag{2.1}$$

$$\Phi_1 = \mathrm{Re}\,(\Phi_0),\ \Phi_2 = \mathrm{Im}\,(\Phi_0),\ \Phi_0 = \Phi_{00} - \Phi_{10}$$

$$\Phi_{00} = \frac{1}{i\omega_2}\left[1 - \frac{a}{a-b}\exp(-i\omega_2 b) + \frac{b}{a-b}\exp(-i\omega_2 a)\right]\exp(i\omega_2 x)$$

$$\Phi_{10} = \Phi_{11} + \Phi_{12} + \Phi_{13},\ \Phi_{11} = (i\omega_2)^{1/3} G \exp\,(i\omega_2 x)$$

$$\Phi_{12} = -\frac{a}{a-b}(i\omega_2)^{1/3} G \exp[i\omega_2(x-b)]$$

$$\Phi_{13} = \frac{b}{a-b}(i\omega_2)^{1/3} G \exp[i\omega_2(x-a)]$$

$$\Omega_1 = i^{1/3}\omega\,\omega_2^{-2/3}$$

$$G = (I_0 - I_1(\Omega_1))\ [\mathrm{Ai}'(\Omega_1) + (i\omega_2)^{4/3}(I_0 - I_1(\Omega_1))]^{-1}$$

$$\mathrm{Ai}'(\Omega_1) = \frac{d\,\mathrm{Ai}(\Omega_1)}{d\Omega_1}$$

$$I_0 = \int_0^{\infty e^{i\varphi}} \mathrm{Ai}(z)\, dz, \quad I_1(\Omega_1) = \int_0^{\Omega_1} \mathrm{Ai}(z)\, dz$$

$$\varphi = (\pi/2 + \arg\omega_2)\ /\ 3$$

where the symbols Re and Im denote the real and imaginary parts of the complex Airy function $\mathrm{Ai}(z)$ which can be specified by the everywhere convergent series

$$\mathrm{Ai}(z) = \frac{1}{3^{2/3}}\left[\sum_{k=0}^{\infty}\frac{z^{3k}}{k!\,3^{2k}\Gamma(k+2/3)} - \frac{z}{3^{1/3}}\sum_{k=0}^{\infty}\frac{z^{3k}}{k!\,3^{2k}\Gamma(k+4/3)}\right] \tag{2.2}$$

which is the generally accepted definition of the Airy function [7, 8]; the definition used in [5, 9] differs from (2.2) by a constant factor. Since formulas (2.1) defining pressure p_1 contain the ratio of Airy functions, they are independent of that factor.

In conformity with (2.1) the calculation of pressure p_1 reduces to the calculation of the integral

$$\int_{-\infty}^{\infty} \Phi_0\, d\omega_2 = \int_{-\infty}^{\infty} \Phi_{00}\, d\omega_2 - \int_{-\infty}^{\infty} \Phi_{10}\, d\omega_2 \tag{2.3}$$

where the first integral in the right-hand side is explicitly determined by

$$\int_{-\infty}^{\infty} \Phi_{00}\, d\omega_2 = \pi\left[\mathrm{sign}\,(x) - \frac{a}{a-b}\,\mathrm{sign}\,(x-b) + \frac{b}{a-b}\,\mathrm{sign}\,(x-a)\right] \tag{2.4}$$

The second integral in the right-hand side of (2.3) is equal to the sum of three integrals of functions Φ_{11}, Φ_{12} and Φ_{13}. Since the integrands Φ_{12} and Φ_{13} differ from Φ_{11} only by constant factors and by values of parameters in the exponent index, their respective integrals are calculated similarly to the integral of Φ_{11}.

Let us calculate the integral of Φ_{11}. From (2.1) we have

$$I_2(\omega, x) = \int_{-\infty}^{\infty} \Phi_{11}\, d\omega_2 = \int_{-\infty}^{\infty} \frac{e^{i\omega_2 x}(i\omega_2)^{1/3}(I_0 - I_1(\Omega_1))}{\mathrm{Ai}'(\Omega_1) + (i\omega_2)^{4/3}(I_0 - I_1(\Omega_1))}\, d\omega_2, \tag{2.5}$$

$$i = e^{i\pi/2}$$

Let us analyze the integrand in (2.5) in the complex plane of the variable ω_2. To separate the single-valued branch we have to make a slit in it. Using the Airy function property

$$|\operatorname{Ai}(z)| \to 0, \quad |z| \to \infty, \quad -\pi/3 \leqslant \arg z \leqslant \pi/3$$

we make the slit from point 0 along the imaginary axis, i.e. $\pi/2 > \arg \omega_2 > -3\pi/2$, then

$$I_0 = \int_0^\infty \operatorname{Ai}(x)\,dx = \frac{3^{1/2}\Gamma(2/3)\,\Gamma(4/3)}{2\pi} \tag{2.6}$$

Let us determine the roots of the integrand denominator, which are poles of the latter; the equation which determines them is conveniently written in terms of the only variable Ω_1

$$F_1(\omega, \Omega_1) = \Omega_1^2 \operatorname{Ai}'(\Omega_1) - \omega^2 (I_0 - I_1)(\Omega_1)) = 0 \tag{2.7}$$
$$-\pi/6 < \arg \Omega_1 < 7\pi/6$$

The constraints imposed on $\arg \Omega_1$ are related to those imposed on $\arg \omega_2$. Having fixed ω, we plot in the complex plane Ω_1 two sets of level curves: $\operatorname{Re}(F_1(\omega, \Omega_1)) = 0$ and $\operatorname{Im}(F_1(\omega, \Omega_1)) = 0$; the intersection points of curves of the different sets determine the roots of Eq. (2.7). A characteristic pattern of level curves

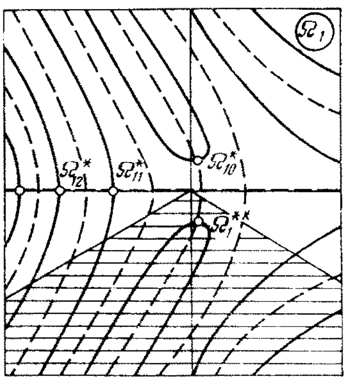

Fig. 2

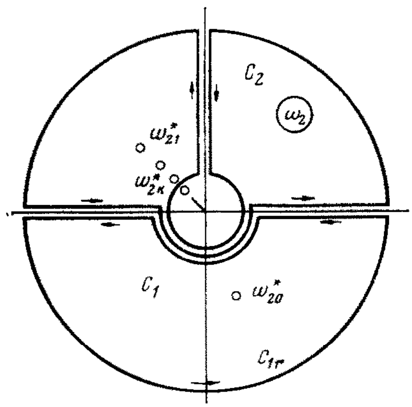

Fig. 3

($\omega = 1$) is shown in Fig. 2, where the solid curves represent lines $\operatorname{Re}(F_1) = 0$, the dash curves the lines $\operatorname{Im}(F_1) = 0$, and the part of the complex plane Ω_1 where the constraint (2.7) on $\arg \Omega_1$ are not satisfied is shown shaded. The roots of Eq. (2.7) are indicated by symbols Ω_1^{**}, Ω_{10}^*, Ω_{11}^*, ...

On the negative part of the real axis we have a denumerable set of roots tending to infinity. In the first quadrant we have one root Ω_{10}^*, and in the fourth quadrant also the single root Ω_1^{**} which must be rejected, since it lies in the "prohibited" part of the plane of variable Ω_1. However root Ω_1^{**} may be used for the convenience of calculations, since it has to be taken into account when passing on the second sheet of a complete Riemann surface of variable ω_2, as will be done in Sect. 3. But no physical meaning can be assigned to the root Ω_1^{**}, since ω_2^{**} with $\arg \omega_2^{**} > \pi/2$ correspond to it, and for such ω_2 equality (2.6) does not hold. On the contrary, roots Ω_{10}^*, Ω_{11}^*, . . . can be readily interpreted. For this we return to the plane ω_2 represented in Fig. 3. The roots Ω_{10}^* and $\Omega_{11}^*, \Omega_{12}^*, \dots$ turn into ω_{20}^* which lies in the fourth quadrant and into ω_{21}^*, ω_{22}^*, . . ., which lie on the segment issuing from the coordinate origin at angle $-5\pi/4$. For considerable ω the first roots Ω_{11}^*, Ω_{12}^*, . . . lie not on the negative axis itself but in its neighborhood. However, as $\omega \to \infty$, the relations $\arg \Omega_{11}^* \to \pi$ and $\arg \Omega_{12}^* \to \pi$, . . . are valid. The result of this is that part of roots ω_{21}^*, ω_{22}^*, . . . have arguments are not equal $-5\pi/4$, but close to that figure.

The equation (2.7) expressed in other variables had already appeared in literature [3, 5]. It occurs in the form of a dispersion relation in investigation of a special kind of perturbations that propagate up- and downstream of the flow. The roots of that equation correspond to some characteristic solutions of (1.6). Thus ω_{20}^* generates a solution that defines the damping of perturbations upstream $(x < 0)$, and when $\omega = 0$ it becomes the solution obtained in [10]. The characteristic functions generated by ω_{21}^*, ω_{22}^*, . . . can be interpreted as perturbations that propagate downstream [6].

As noted in [6], it is convenient, when calculating integral (2.5), to separate in the plane ω_2 two contours. For $x < 0$ we select the integration path C_1 (Fig. 3) in the lower half-plane consisting of the segment of real axis from r to $-r$ bypassing point O and of the arc of circle of radius r. For $x > 0$ we select C_2 as the integration path which basically lies in the upper half-plane and consists of a segment of the real axis from $-r$ to r bypassing point O and of the arcs of circles of radius r, and the two edges of the slit along the imaginary axis, bypassing point O. Applying the Cauchy theorem on residues to the integral along the contour C_1 and making the radius of the large semicircle r approach infinity, and that of small semicircle to approach zero, in the case of $x < 0$ we obtain

$$\int_{-\infty}^{\infty} \Phi_{11}\, d\omega_2 = -2\pi i \operatorname{res} \Phi_{11}(\omega_{20}^*) = B(\omega, \omega_{20}^*) \exp(i\omega_{20}^* x) \tag{2.8}$$

$$B = -3\pi i \frac{I_0 - I_1(\Omega_{10}^*)}{2i(I_0 - I_1(\Omega_{10}^*)) + \Omega_{10}^*(i - \omega^2/\omega_{20}^{*2}) \operatorname{Ai}(\Omega_{10}^*)}$$

The theorem on residues can be also applied to contour C_2 with the only difference that the circle enveloping point O is to be contracted not continuously but discretely, drawing it each time between the adjacent poles ω_{2k}^* and ω_{2k+1}^*. The series in residues $\operatorname{res} \Phi_{11}(\omega_{2k})$ is rapidly convergent, since $|\operatorname{res} \Phi_{11}(\omega_{2k})| = O(k^{-3})$ when $k \to \infty$. As the result in the case of $x > 0$ we have

$$\int_{-\infty}^{\infty} \Phi_{11}\, d\omega_2 = 2\pi i \sum_{k=1}^{\infty} \operatorname{res} \Phi_{11}(\omega_{2k}^*) + I_3(\omega, x) + I_4(\omega, x) \tag{2.9}$$

where I_3 and I_4 are integrals of Φ_{11} taken along the edges of the slit. Each term in the right-hand side of (2.9) may be considered as a running wave of varying amplitude, as was done in [11].

Formula (2.8) is quite convenient for use on a computer, which cannot be said about formula (2.9). It presents particular difficulties in the computation of integrals

I_3 and I_4. We shall choose a method of computation that is not based on formula (2.9). We use the C_1 contour also for $x > 0$, denoting its part consisting of the semicircle of radius r by C_{1r}, and apply the theorem on residues (with decreasing radius the integral along the small semicircle of contour C_1 approaches zero independently of the sign of x)

$$\int_{-r}^{r} \Phi_{11}\, d\omega_2 = \int_{C_{1r}} \Phi_{11}\, d\omega_2 - 2\pi i \operatorname{res} \Phi_{11}(\omega_{20}^*) \tag{2.10}$$

For $r \gg 1$ in the computation of the integral along the C_{1r} contour the quantity $|\Omega_1| = \omega / r^{2/3} \ll 1$. For such values of Ω_1 we replace the Airy function by series (2.2), and write the fraction in the integrand of (2.5) in the form (see (2.1))

$$G(\omega, \Omega_1) = \Omega_1^2 \Big(\sum_{k=0}^{\infty} c_{1k} \Omega_1^k \Big) \Big(\sum_{k=0}^{\infty} c_{2k} \Omega_1^k \Big)^{-1} \tag{2.11}$$

where the coefficients c_{1k} and c_{2k} are determined using the series expansion of the integral and the derivative of the Airy function, which are obtained by term-by-term integration and differentiation of series (2.2). Coefficients c_{2k} depend on ω. We divide the series in the right-hand side of (2.11) and obtain

$$G(\omega, \Omega_1) = \Omega_1^2 (c_{g,0} + c_{g,1}\Omega_1 + \ldots + c_{g,k}\, \Omega_1^k + \ldots) \tag{2.12}$$

Note that in this formula $c_{g,0} = -1 / \omega^2$ and $c_{g,1} = 0$. We substitute expansion (2.12) into integral (2.10) and pass to the variable ω_2 (we recall that as shown in (2.5) $i = e^{i\pi/2}$)

$$\int_{C_{1r}} \Phi_{11}\, d\omega_2 = \int_{C_{1r}} \left(\frac{i\omega^2}{\omega_2} c_{g,\,0} + \ldots + \frac{i^{1+k/3} \omega^{2+k}}{\omega_2^{1+2k/3}} c_{g,\,k} + \ldots \right) \times \tag{2.13}$$
$$\exp(i\omega_2 x)\, d\omega_2$$

Integration of each term of series (2.13) can be carried out analytically, to do this it is convenient to separate three groups of terms two of which depend on the Γ-function. Passing to limit with $r \to \infty$ we obtain

$$I_C(\omega, x) = \lim_{r\to\infty} \int_{C_{1r}} \Phi_{11}\, d\omega_2 = -2\pi\omega^2 \sum_{k=0}^{\infty} (i\omega)^{3k} x^{2k} \frac{c_{g,\,3k}}{(2k)!} - \tag{2.14}$$
$$i\sqrt{3}\,\Gamma\left(\frac{1}{3}\right) \omega^3 x^{2/3} \sum_{k=1}^{\infty} (i\omega)^{3k} x^{2k} \frac{c_{g,\,3k+1}}{{}^2\!/_3 \cdot {}^5\!/_3 \ldots (2k + {}^2\!/_3)} +$$
$$\sqrt{3}\,\Gamma\left(\frac{2}{3}\right) \omega^4 x^{4/3} \sum_{k=0}^{\infty} (i\omega)^{3k} x^{2k} \frac{c_{g,\,3k+2}}{{}^1\!/_3 \cdot {}^4\!/_3 \ldots (2k + {}^4\!/_3)}$$

The lower boundary of the index k of the second sum in (2.14) is unity, since as noted above $c_{g,1} = 0$ this equality ensures the vanishing of the derivative dI_C / dx when $x = 0$. This method of computation of the integral $I_C(\omega, x)$ is convenient when $\omega \geqslant 1$. For $\omega < 1$ formula (2.11) must be expanded in series using the variable $\Omega_2 = \Omega_1 / \omega$. This evidently results in the change of coefficients $c_{g,k}$ (of course, as previously, $c_{g,1} = 0$). A series similar to (2.14) may in turn be obtained from (2.14) be setting in it $\omega = 1$. The selection of expansion in the variable Ω_1 or Ω_2 is affected by that coefficients $c_{g,k}$ have to be computed prior to their substitution to series (2.14). Hence for a specified ω it is desirable to use a modification of the expansion such that would ensure a not too rapid increase of coefficients $c_{g,k}$ with increasing number k. Using (2.8) and (2.14) from formula (2.10) we obtain

$$\int_{-\infty}^{\infty} \Phi_{11}\, d\omega_2 = B(\omega, \omega_{20}^*) \exp(i\omega_{20}^* x) + I_C(\omega, x) \tag{2.15}$$

We extend the notation $I_C(\omega, x)$ introduced in (2.14) to the new independent variables, and shall use the symbol $I_C(\omega, x - x_0)$ to define series (2.14) in which $x - x_0$ has been substituted for x. This makes it possible to write

$$\int_{-\infty}^{\infty} \Phi_{12}\, d\omega_2 = -\frac{a}{a-b}\left[B(\omega, \omega_{20}^*) \exp(i\omega_{20}^*(x-b)) + I_C(\omega, x-b)\right] \tag{2.16}$$

$$\int_{-\infty}^{\infty} \Phi_{13}\, d\omega_2 = \frac{b}{a-b}\left[B(\omega, \omega_{20}^*) \exp(i\omega_{20}(x-a)) + I_C(\omega, x-a)\right]$$

Finally, substituting (2.4), (2.15), and (2.16) into formula (2.3), for the integral of Φ_0 which according to (2.1) defines pressure we obtain

$$\begin{aligned}\int_{-\infty}^{\infty} \Phi_0\, d\omega_2 = B(\omega, \omega_{20}^*)\Big[1 - \frac{a}{a-b}\exp(-i\omega_{20}^* b) + {}\\ \frac{b}{a-b}\exp(-i\omega_{20}^* a)\Big]\exp(i\omega_{20}^* x) + \theta(x)\left[2\pi - I_C(\omega, x)\right] - {}\\ \frac{a}{a-b}\theta(x-b)\left[2\pi - I_C(\omega, x-b)\right] + {}\\ \frac{b}{a-b}\theta(x-a)\left[2\pi - I_C(\omega, x-a)\right]\end{aligned} \tag{2.17}$$

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

The use of formula (2.17) on a computer does not present any difficulties, and the computation time which is mainly consumed by the calculation of the segment of series (2.14) with the superscript limit $k \leqslant 15$.

The dependence of pressure p_1 on x for a triangle of dimensions $b = 1$ and $a = 2$ oscillating with frequency $\omega = 1$ for instants of time $t = 0, T/8, T/4, 3T/8$, where $T = 2\pi/\omega$, is shown in Fig.4 by curves I—IV, respectively. The derivative of pressure with respect to x at points $x = 0$ b, and a is continuous but, as implied by the expansion (2.14), the second derive $\partial^2 p / \partial x^2$ has at these points a discontinuity of the second kind, that is related to the discontinuity of derivative $\partial y_w / \partial x$.

The graphs of dependence of pressure amplitude of x are shown in Fig. 5

$$A(p_1) = \frac{1}{\pi}\left[(\mathrm{Re}(\Phi_0))^2 + (\mathrm{Im}(\Phi_0))^2\right]^{1/2}$$

for a triangle, with same dimensions as for Fig. 4, oscillating with frequencies $\omega = 0$ (stationary triangle); 1; 3. If flow over stationary triangle has the point x, in which $A(p_1) = 0$, then there will not be such point for flow over oscillating triangle.

3. Asymptotics of pressure when $x \to \infty$ and $\omega \to \infty$. The behavior of pressure as $x \to -\infty$ is obtained from the analysis of formula (2.17). Since $\operatorname{Im} \omega_{20}^* < 0$, the pressure tends to zero in conformity with the exponential law $\exp(-\operatorname{Im}(\omega_{20}^*)x)$. When $x \to \infty$ formula (2.9) makes it possible to assert that $I_2(\omega, x) \to 0$ since the integrands of integrals $I_3(\omega, x)$ and $I_4(\omega, x)$ are exponentially decreasing functions, and the series consists of terms each of which decreases according to an exponential law. The determination of the asymptotics of integrals I_3 and I_4 does not present any difficulties, but the determination of asymptotics of series (2.9) is fraught with difficulties. Note also that as $x \to \infty$, formula (2.15) cannot be used for computations, since it results in the remainder of

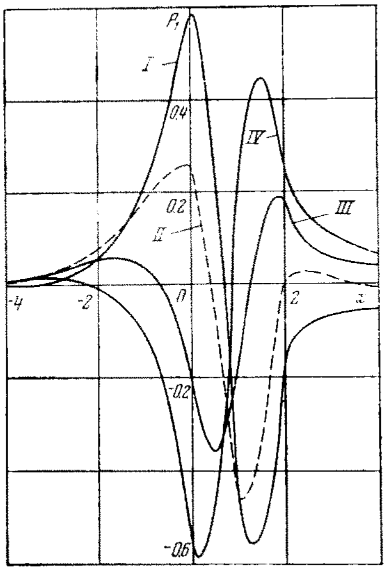

Fig. 4

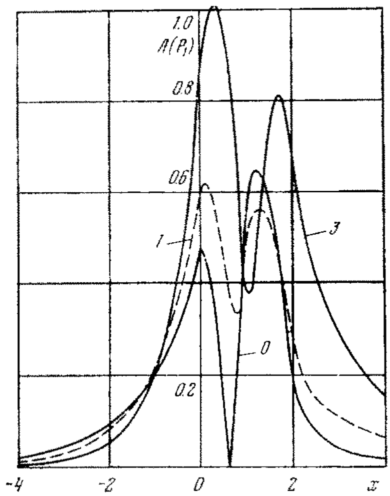

Fig. 5

two quantities both approaching infinity.

The analysis of integral (2.5) when $x \to \infty$ is again carried out in the complex plane ω_2. As the integration path we select contour C_3 consisting of a segment of the real axis bypassing point O, two arcs C_{3r1} and C_{3r2}, and two segments C_{31} and C_{32} bypassing point O. Let the angle of inclination α of segment C_{31} to the real axis be less than $\pi / 4$ (see Fig. 6). Making the radii of arcs C_{3r1} and C_{3r2} approach infinity and the radius of the bypass of point O tend to zero, for $x > 0$ we obtain

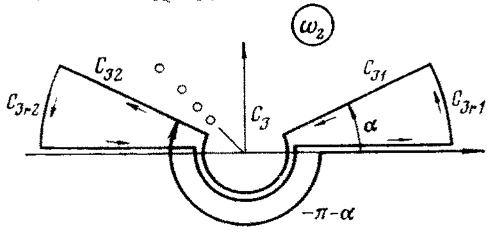

Fig. 6

$$\int_{-\infty}^{\infty} \Phi_{11}\, d\omega_2 = -\left(\int_{C_{31}} + \int_{C_{32}}\right) \Phi_{11}\, d\omega_2$$

The integrals C_{31} and C_{32} with the respective substitution of variables assume the form

$$C_{31}:\ \omega_2 = e^{i\alpha}q,\ \Omega_1 \equiv \Omega_{11} = e^{i(\pi/6-2\alpha/3)}\,\omega q^{-2/3} \tag{3.1}$$

$$\int\limits_{C_{31}} \Phi_{11}\,d\omega_2 = -\int\limits_0^\infty (I_0 - I_1(\Omega_{11}))\exp[ixq\cos\alpha -$$
$$xq\sin\alpha + i(\pi/6 + 4\alpha/3)]\,[\mathrm{Ai}'(\Omega_{11}) +$$
$$e^{i(2\pi/3+4\alpha/3)}q^{4/3}(I_0 - I_1(\Omega_{11}))]^{-1}q^{1/3}\,dq$$

$$C_{32}:\ \omega_2 = e^{-i(\pi+\alpha)}q,\ \Omega_1 \equiv \Omega_{12} = e^{i(5\pi/6+2\alpha/3)}\omega q^{-2/3} \tag{3.2}$$

$$\int\limits_{C_{32}} \Phi_{11}\,d\omega_2 = \int\limits_0^\infty (I_0 - I_1(\Omega_{12}))\exp\Big[-ixq\cos\alpha - xq\sin\alpha -$$
$$i\left(\frac{7\pi}{6} + \frac{4\alpha}{3}\right)\Big]\,[\mathrm{Ai}'(\Omega_{12}) + e^{-i(2\pi/3+4\alpha/3)}q^{4/3}(I_0 - I_1(\Omega_{12}))]^{-1}q^{1/3}\,dq$$

We investigate integral (3.1) in the upper half-plane formed by the first and second sheet of the complete Riemann surface. We have now for $\arg\omega_2$ the constraint $\pi > \arg\ \omega_2 > 0$. In the second quadrant we have the first order pole ω_2^{**} which corresponds to the root of the variance relation Ω_1^{**} (see Fig. 2) and, as shown by the analysis of function $\omega_2^{**}(\omega)$, the inequality $\arg\ \omega_2^{**} > 3\pi/4$ is correct. If the ray issuing from the coordinate origin at angle β is taken as the integration path in (3.1), the integral (3.1) is independent of angle β when

$$0 < \beta < \arg\ \omega_2^{**} \tag{3.3}$$

Taking into account (3.3) and setting $\beta = \pi - \alpha$, we obtain for α the inequality

$$\pi - \arg\ \omega_2^{**} < \alpha < \pi/4 \tag{3.4}$$

The set of α that satisfies inequality (3.4) is nonempty, since, as indicated previously, $\arg\omega_2^{**} > 3\pi/4$. Note that if α is chosen not from the range (3.4) but from the wider range $0 < \alpha < \pi/4$, the final result remains unchanged, although it is then necessary to take into account the residue $\operatorname{res}\Phi_{11}(\omega_2^{**})$.

Setting $\beta = \pi - \alpha$ and carrying out integration with respect to C_{31}', we rewrite integral (3.1) in the form

$$C_{31}':\ \omega_2 = e^{i(\pi-\alpha)}q,\ \Omega_1 \equiv \Omega_{13} = e^{-i(\pi/2-2\alpha/3)}\,\omega q^{-2/3} \tag{3.5}$$

$$\int\limits_{C_{31}} \Phi_{11}\,d\omega_2 = \int\limits_{C_{31}'} \Phi_{11}\,d\omega_2 = -\int\limits_0^\infty \exp\Big[\varphi_1(x, q, \alpha) +$$
$$i\left(\frac{3\pi}{2} - \frac{4\alpha}{3}\right)\Big]\,(I_0 - I_1(\Omega_{13}))\,[\mathrm{Ai}'(\Omega_{13}) +$$
$$e^{-i4\alpha/3}q^{4/3}(I_0 - I_1(\Omega_{13}))]^{-1}q^{1/3}\,dq$$
$$\varphi_1(x,\ q,\ \alpha) = -ixq\cos\alpha - xq\sin\alpha$$

Combining integrals (3.2) and (3.5) we obtain

$$\int\limits_{-\infty}^\infty \Phi_{11}\,d\omega_2 = -ie^{-i4\alpha/3}\int\limits_0^\infty e^{\varphi_1(x,\,q,\,\alpha)} \tag{3.6}$$

$$[\mathrm{Ai}'(\Omega_{12})(I_0 - I_1(\Omega_{13})) + e^{i\pi/3}\mathrm{Ai}'(\Omega_{13})(I_0 - I_1(\Omega_{12}))][\mathrm{Ai}'(\Omega_{13}) + e^{-i4\alpha/3}q^{4/3}(I_0 - I_1(\Omega_{13}))]^{-1}[\mathrm{Ai}'(\Omega_{12}) + e^{-i(2\pi/3+4\alpha/3)}q^{4/3}(I_0 - I_1(\Omega_{12}))]^{-1}q^{1/3}\,dq$$

According to [8] the Airy function satisfies the relation

$$\mathrm{Ai}(z) + e^{-2\pi i/3}\mathrm{Ai}(e^{-2\pi i/3}z) + e^{-4\pi i/3}\mathrm{Ai}(e^{-4\pi i/3}z) = 0 \tag{3.7}$$

Differentiation and integration in (3.7) yields the corollaries

$$\mathrm{Ai}'(z) + e^{-2\pi i/3}\mathrm{Ai}'(e^{-4\pi i/3}z) + e^{-4\pi i/3}\mathrm{Ai}'(e^{-2\pi i/3}z) = 0 \tag{3.8}$$

$$\int_0^z \mathrm{Ai}(z_1)\,dz_1 + \int_0^{ze^{-2\pi i/3}} \mathrm{Ai}(z_1)\,dz_1 + \int_0^{ze^{-4\pi i/3}} \mathrm{Ai}(z_1)\,dz_1 = 0$$

We apply relations (3.8) to the expression in brackets of the numerator of fraction (3.6) and obtain

$$\int_{-\infty}^{\infty} \Phi_{11}\,d\omega_2 = -ie^{-i4\alpha/3}\int_0^{\infty} e^{\varphi_1(x,\,q,\,\alpha)}G_1(\omega, q, \alpha)\,q^{1/3}\,dq \tag{3.9}$$

$$G_1(\omega, q, \alpha) = [3e^{i\pi/3}I_0\mathrm{Ai}'(e^{-4\pi i/3}\Omega_{12}) - e^{i\pi/3}\mathrm{Ai}'(e^{-4\pi i/3}\Omega_{12})\times (I_0 - I_1(e^{-2\pi i/3}\Omega_{12})) + e^{-i\pi/3}\mathrm{Ai}'(e^{-2\pi i/3}\Omega_{12})(I_0 - I_1(e^{-4\pi i/3}\Omega_{12}))]\times [\mathrm{Ai}'(\Omega_{13}) + e^{-i4\alpha/3}q^{4/3}(I_0 - I_1(\Omega_{13}))]^{-1}[\mathrm{Ai}'(\Omega_{12}) + e^{-i(2\pi/3+4\alpha/3)}q^{4/3}(I_0 - I_1(\Omega_{12}))]^{-1}q^{1/3}\,dq$$

Formula (3.9) is basic for the investigation of the asymptotic behavior of pressure as $x \to \infty$. Note that the integral in the right-hand side of (3.9) is presented in a form convenient for calculating the asymptotics by the saddle-point method. We divide the integration interval from 0 to ∞ in two subintervals: from 0 to ε_1, where $\varepsilon_1 \ll 1$, and from ε_1 to ∞. It can be shown that the integral over the second interval is of the order of $\exp(-\varepsilon_1 x \sin\alpha)$ when $x \to \infty$. Let us consider the integral from 0 to ε_1 in which the inequalities $|\Omega_{12}| = |\Omega_{13}| \geqslant \omega/\varepsilon_1^{2/3} \gg 1$ are valid for Ω_{12} and Ω_{13}. Using the asymptotic expansion of the Airy function in the neighborhood of an infinitely distant point [8]

$$\mathrm{Ai}(z) \sim \frac{1}{2}z^{-1/4}\exp\left(-\frac{2}{3}z^{3/2}\right)\sum_{k=0}^{\infty} a_k z^{-3k/2} \tag{3.10}$$

$$|z| \to \infty, \quad |\arg z| < \pi$$

where a_k are real coefficients dependent on the ordinal number k [8]. Using formula (3.10) we obtain for the derivative and the integral of Airy's function the asymptotic expansion

$$\mathrm{Ai}'(z) \sim -\frac{1}{2}z^{1/4}\exp\left(-\frac{2}{3}z^{3/2}\right)\sum_{k=0}^{\infty} b_k z^{-3k/2} \tag{3.11}$$

$$\int_0^z \mathrm{Ai}(z_1)\,dz_1 \sim \int_0^{\infty}\mathrm{Ai}(x)\,dx - \frac{1}{2}z^{-3/4}\exp\left(-\frac{2}{3}z^{3/2}\right)\sum_{k=0}^{\infty} c_k z^{-3k/2}$$

$$|z| \to \infty, \quad |\arg z| < \pi$$

where the real constants b_k and c_k depend on the ordinal number k, and $a_0 = b_0 = c_0 = 1$. Let us substitute for the derivatives and integrals of the Airy functions their asymptotics determined by (3.11). This can always be done for α that satisfies condition (3.4), since the absolute values of arguments of expressions $e^{-4\pi i/3}\,\Omega_{12}$, $e^{-2\pi i/3}\,\Omega_{12}$, Ω_{12}, and Ω_{13} are smaller than π. As in Sect. 2 (the passage from (2.11) to (2.12), we carry out the division of polynomials and obtain

$$G_1(\omega, q, \alpha) \sim 6I_0 e^{\pi i/3}\Omega_{12}^{-1/4}\exp\left(\frac{2}{3}\Omega_{12}^{3/2}\right)\sum_{k=0}^{\infty} c_{g1,k}\,\Omega_{12}^{-3k/2} \tag{3.12}$$

We substitute expansion (3.12) into the right-hand side of (3.9) where integration is carried out from 0 to ε_1, and investigate the term k of that expansion

$$I_k(\omega, x, \alpha) = 6iI_0 c_{q1,k}\,\omega^{-1/4-3k/2}\exp\left[i\pi\left(1/8 - 5k/4\right) - i\alpha\left(3/2 + k\right)\right] I_{k1} \tag{3.13}$$

$$I_{k1}(\omega, x, \alpha) = \int_0^{\varepsilon_1}\exp\left[\varphi_2(\omega, x, q, \alpha)\right] q^{1/2+k}\,dq$$

$$\varphi_2 = -xq\sin\alpha - {}^2/_3\,\omega^{3/2}q^{-1}\cos\left(\pi/4 + \alpha\right) - i\left[xq\cos\alpha + {}^2/_3\,\omega^{3/2}q^{-1}\sin\left(\pi/4 + \alpha\right)\right]$$

We carry out in the integral I_{k1} the substitution of variables $q = q_1\sqrt{x}$, which brings the upper limit of integration to $\varepsilon_1\sqrt{x}$. It can be shown that the change of the upper limit of integration to ∞ introduces an error of the order of $\exp(-\varepsilon_1 x \sin\alpha)$. We have

$$I_{k1} = x^{-(3/2+k)/2}\int_0^{\infty}\exp\left[-\sqrt{x}\,\varphi_3(\omega, q_1, \alpha)\right] q_1^{1/2+k}\,dq_1 \tag{3.14}$$

$$\varphi_3 = q_1\sin\alpha + {}^2/_3\,\omega^{3/2}q_1^{-1}\cos\left(\pi/4 + \alpha\right) + i\left[q_1\cos\alpha + {}^2/_3\,\omega^{3/2}q_1^{-1}\sin\left(\pi/4 + \alpha\right)\right]$$

To evaluate function I_{k1} when $x \to \infty$ we can use the saddle-point method. This is, however, unnecessary here, since for this type of functions $\varphi_3(\omega, q_1, \alpha)$ the integral in the right-hand side of (3.14) has been thoroughly analyzed. We have (see [12])

$$I_{k1} = x^{-(3/2+k)/2}\,2\left[{}^2/_3\,\omega^{3/2}e^{i(-\pi/4+2\alpha)}\right]^{3/4+k/2} K_{k+3/2}\left(2^{3/2}3^{-1/2}\omega^{3/4}e^{3\pi i/8}x^{1/2}\right) \tag{3.15}$$

$$K_{k+3/2}(z) = \sqrt{\frac{\pi}{2z}}\,e^{-z}\sum_{m=0}^{k+1}\frac{(k+1+m)!}{m!\,(k+1-m)!\,(2z)^m} \tag{3.16}$$

In turn, substituting (3.15) into (3.13) and (3.13) into (3.9), when $x \to \infty$ we obtain the asymptotics of integral (3.9)

$$\int_{-\infty}^{\infty}\Phi_{11}\,d\omega_2 \sim I_{\mathrm{as}}(\omega, x) \equiv 12i\left(\frac{2}{3}\right)^{3/4} I_0\,\omega^{1/4}x^{-3/4}e^{-\pi i/16}\times \tag{3.17}$$

$$\sum_{k=0}^{\infty} c_{q1,k}\left(\frac{2}{3}\right)^{k/2} e^{-11\pi ik/8}\omega^{-3k/4}x^{-k/2}K_{k+3/2}\left(2^{3/2}3^{-1/2}\omega^{3/4}e^{3\pi i/8}x^{1/2}\right)$$

Using the equality $c_{g1,0} = 1$ and calculating $K_{3/2}$ by formula (3.16) for the principal term of asymptotics (3.17) we obtain

$$\int_{-\infty}^{\infty} \Phi_{11}\, d\omega_2 \sim 2i\sqrt{6\pi}\, I_0 e^{-\pi i/4} x^{-1} \exp\left(-2^{3/2} 3^{-1/2} \omega^{3/4} e^{3\pi i/8} x^{1/2}\right) \tag{3.18}$$

As expected, asymptotics (3.17) is independent of angle α. We extend the notation $I_{as}(\omega, x)$ to the new variables, and shall use the symbol $I_{as}(\omega, x - x_0)$ to denote series (3.17) in which $x - x_0$ has been substituted for x. This makes it possible when $x \to \infty$ to write the asymptotics of the integral of Φ_0 as

$$\int_{-\infty}^{\infty} \Phi_0\, d\omega_2 \sim -\left[I_{as}(\omega, x) - \frac{a}{a-b} I_{as}(\omega, x - b) + \frac{b}{a-b} I_{as}(\omega, x - a)\right] \tag{3.19}$$

Formulas (2.17) and (3.19) complement each other: the first is a convergent expansion in variable x of the integral of Φ_0 at zero $(x > 0)$, while the second is a divergent asymptotic expansion at infinity. In spite of the divergence of series (3.19) it is convenient for computations on a computer with a reduced number of terms in the series.

Pressure perturbations induced by the oscillating triangle tend to dampen as $x \to -\infty$ according to the exponential law (2.8) with the exponent proportional to x, as $x \to \infty$ the damping may also occur in conformity with the exponential law (3.18). However, in that case the exponent is proportional to $-\sqrt{x}$ (presence in the coefficient of the exponent of power x is not taken into account).

Let us compare these results with the asymptotics of pressure perturbation induced by the triangle at rest, $\omega = 0$ As $x \to -\infty$, the damping is defined, as previously, by formula (2.8), but at a slower rate, since $\mathrm{Im}\, \omega_{20}^*\ (\omega = 0) > \mathrm{Im}\, \omega_{20}^*\ (\omega \neq 0)$. It can be shown that, as $x \to \infty$ the properties of asymptotics of p_1 change: pressure obeys the power law $p_1 \sim x^{-10/3}$. Hence, pressure perturbations induced by the oscillating triangle dampen more rapidly as $x \to \pm\infty$, than a perturbation induced by a triangle at rest. This is particularly prominent downstream [of the triangle], as $x \to \infty$, where the nature of damping for $\omega \neq 0$ is exponential, and for $\omega = 0$ of the power type.

In the investigation of perturbation asymptotics of pressure p_1 as $\omega \to \infty$ we use integral (2.5) which for $x < 0$ is obtained in explicit form as shown in (2.8). Using asymptotics (3.11) we write the solution of the dispersed equation (2.7) in the form of series [3]

$$\omega_{20}^* = e^{-\pi i/4} \omega^{1/2} + \tfrac{1}{2} e^{-3\pi i/4} \omega^{-1/2} + \ldots \tag{3.20}$$

We determine the quantity $B(\omega, \omega_{20}^*)$ as $\omega \to \infty$ and $x < 0$ using (3.20) and (3.11), and obtain the principal term of expansion (2.5)

$$\int_{-\infty}^{\infty} \Phi_1 d\omega_2 \sim -\pi \exp\left(\omega^{1/2} x e^{\pi i/4}\right) \tag{3.21}$$

The method of constructing the asymptotics of integral (2.5) when $\omega \to \infty$ and

$x > 0$ is the same as used for deriving the asymptotics of this integral when $x \to \infty$, except for the selection of angle α. When $\omega \to \infty$ for the root of (2.7) on the second sheet of the Riemann surface we have

$$\omega_2^{**} = e^{3\pi i/4}\omega^{1/2} + \frac{1}{2} e^{-3\pi i/4}\omega^{-1/2} + \ldots \tag{3.22}$$

which implies that at the limit, as $\omega \to \infty$, the inequality (3.4) cannot be satisfied, since $\arg\ \omega_2^{**} \to 3\pi / 4$. This implies that α is to be selected from the range

$$0 < \alpha < \pi - \arg\ \omega_2^{**}$$

As the result, we write the basic formula for estimate of integral (2.5) as

$$\int_{-\infty}^{\infty} \Phi_{11} d\omega_2 = -ie^{-i4\alpha/3} \int_0^{\infty} e^{\varphi_1} G_1(\omega, q, \alpha)\, q^{1/3} dq + 2\pi i \operatorname{res} \Phi_{11}(\omega_2^{**}) \tag{3.23}$$

$$\varphi_1 = -ixq\cos\alpha - xq\sin\alpha$$

Although the method of evaluation of the integral in the right-hand side of (3.23) when $\omega \to \infty$ somewhat differs from that used for evaluating integral (3.9) when $x \to \infty$, the result is the same in both cases: the integral approaches zero proportionally as $\exp(-2^{3/2}3^{-1/2}\omega^{3/4} e^{8\pi i/3} x^{1/2})$. But the second term in the right-hand side of (3.23) approaches zero as $\exp(-\omega^{1/2}e^{\pi i/4}x)$. Hence by retaining in the right-hand side of (3.23) only the second principal term we have

$$\int_{-\infty}^{\infty} \Phi_{11} d\omega_2 \sim -\pi \exp(-\omega^{1/2}e^{\pi i/4}x), \quad \omega \to \infty,\ x > 0 \tag{3.24}$$

The expansion of integrals of Φ_{12} and Φ_{13}, as $\omega \to \infty$ are analogous.

Let us consider once again the integral of Φ_0 and, in accordance with (2.1), represent it in the form

$$\int_{-\infty}^{\infty} \Phi_0 d\omega_2 = \int_{-\infty}^{\infty} \Phi_{00} d\omega_2 - \int_{-\infty}^{\infty} \Phi_{11} d\omega_2 - \int_{-\infty}^{\infty} \Phi_{12} d\omega_2 - \int_{-\infty}^{\infty} \Phi_{13} d\omega_2 \tag{3.25}$$

The first integral in the right-hand side of (3.25) whose explicit form appears in (2.4) is independent of ω; as $\omega \to \infty$ the second integral is, according to (3.21) and (3.24), nonzero only in a small neighborhood of point $x = 0$, and, similarly, the third and fourth integrals are nonzero in the neighborhoods of points $x = b$ and $x = a$, respectively. Hence, when $\omega \to \infty$, the basic part is played by the discontinuous integral (2.4). The integrals of functions Φ_{11}, Φ_{12}, and Φ_{13} make possible only the continuous joining of discontinuities in (2.4) at points $x = 0$, $x = b$, $x = a$. Since the imaginary part of integral (3.25) is zero as $\omega \to \infty$, pressure p_1 is also determined by formula (2.4). The dependence of p_1 [on x] with $\omega \to \infty$ is shown in Fig. 7 for instants of time $t = 0,\ T / 8,\ T / 4,\ 3T / 8$, where $T = 2\pi / \omega$, by curves I—IV, respectively.

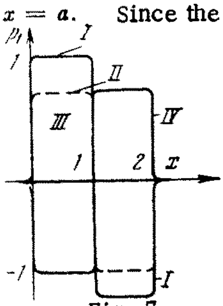

Fig. 7

Let us show that the derived dependence of pressure on x and t as $\omega \to \infty$ is exactly the same as in inviscid supersonic stream at slow wall oscillations (1.4). For this we rewrite relation (1.4) in dimensional form

$$y_w^* = \varepsilon^2 C^{1/4} \lambda^{1/2} (M_\infty^2 - 1)^{1/4} \sigma f_{10}(x^*) \cos \omega t' \tag{3.26}$$

using (1.1) and retaining t as the dimensionless time. In this equation function $f_{10}(x^*)$ defines the triangular form (1.5) with parameters a^* and b^* related to a and b in the same way as x^* to x from formula (1.1).

Let us determine the pressure induced by wall oscillations (3.26) in the region of characteristic dimensions $x^* = O(\varepsilon^3)$, $y^* = O(\varepsilon^3)$ at characteristic time $t^* = O(\varepsilon^2)$ in an inviscid supersonic stream. We have

$$p^* = p_\infty^* + \rho_\infty^* v_\infty^{*2} \varepsilon^2 C^{1/4} \lambda^{1/2} (M_\infty^2 - 1)^{-1/4} \sigma \times$$
$$\left[\operatorname{sign} x^* - \frac{a^*}{a^* - b^*} \operatorname{sign}(x^* - b^*) + \frac{b^*}{a^* - b^*} \operatorname{sign}(x^* - a^*)\right] \cos \omega t'$$

Such coincidence is due to that when $\omega \to \infty$, the dependence on time must allow for the basic boundary layer thickness $(y \sim \varepsilon^4)$. This problem can also be solved by using the fundamental concept of Prandtl according to which pressure is to be determined by solving the external inviscid problem.

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